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Rayleigh–Taylor Instability of Two Superposed Conducting Fluids in the Presence of a Variable Magnetic Field and Suspended Particles

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The instability of two superposed fluids in the presence of suspended particles is studied. It is found that the system is unstable for sufficiently long waves, however large the applied magnetic field may be, provided the initial configuration is top-heavy density wise. Also, the potentially stable system remains stable for the case of two superposed fluids in the presence of a variable magnetic field and suspended particles.

1. INTRODUCTION

The instability of the plane interface separating two fluids when one is accelerated towards the other or when one is superposed over the other has been studied by several authors, and Chandrasekhar [1] has given a detailed account of these investigations. Roberts [2] extended the analysis to the case of two fluids of equal kinematic viscosities while Gerwin [3] has studied the case of compressible streaming fluids. Kruskal and

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Schwarzschild [4] have considered the stability of an inviscid plasma of infinite conductivity supported against gravity by a horizontal magnetic field. Hide [5] studied the case of a viscous conducting fluid with a transverse magnetic field and found that the magnetic field considerably stabilizes the configuration and it is possible to have oscillatory motion in the presence of a magnetic field even if the configuration is thoroughly unstable.

The effect of suspended particles on the stability of superposed fluids might be of industrial and chemical engineering importance. Further motivation for this study is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance. Scanlon and Segel [6] considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. Sharma *et al.* [7] studied the effect of suspended particles on the onset of Bénard convection in hydromagnetics. The effect of suspended particles was found to destabilize the layer whereas the effect of a magnetic field was stabilizing. In all of the above studies, the magnetic field has been considered to be uniform. Sharma and Thakur [8] investigated Rayleigh–Taylor instability of a partially ionized medium in the presence of a variable horizontal magnetic field. The present paper is devoted to the consideration of Rayleigh–Taylor instability of two superposed conducting fluids in the presence of a variable horizontal magnetic field and suspended particles.

We consider a static state in which an incompressible fluid-particle layer of variable density is arranged in horizontal strata and the pressure p and the density ρ are functions of the vertical coordinate z only. The character of the equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution. The fluid is under the action of gravity $\mathbf{g}(0, 0, -g)$ and the variable horizontal magnetic field $\mathbf{H}(H(z), 0, 0)$. The particles are assumed to be non-conducting.

2. Basic Equations

Let ρ, μ, p, and $\mathbf{u}(u, v, w)$ denote, respectively, the density, the viscosity, the pressure, and the velocity of the pure gas; $\mathbf{V}(\bar{x}, t)$ and $N(\bar{x}, t)$ denote the velocity and number density of the particles, respectively. $K = 6\pi\mu\eta$, where η is the particle radius, is a constant and $\bar{x} = (x, y, z)$. Then the equations of motion and continuity for the gas and Maxwell's equations are

$$\rho\left[\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}\right]$$
$$= -\nabla p+\rho\mathbf{g}+\mu\nabla^2\mathbf{u}+KN(\mathbf{V}-\mathbf{u})$$
$$+\frac{\mu_e}{4\pi}[(\nabla\times\mathbf{H})\times\mathbf{H}]+\left(\frac{\partial w}{\partial \bar{x}}+\frac{\partial \mathbf{u}}{\partial z}\right)\frac{d\mu}{dz}, \tag{2.1}$$

$$\nabla\cdot\mathbf{u}=0, \tag{2.2}$$

$$\frac{\partial \mathbf{H}}{\partial t}=\nabla_{X}(\mathbf{u}\times\mathbf{H}), \tag{2.3}$$

$$\nabla\cdot\mathbf{H}=0, \tag{2.4}$$

where μ_e, the magnetic permeability, is assumed to be constant and the fluid is assumed to be infinitely conducting.

Since the density of a particle moving with the fluid remains unchanged, we have

$$\frac{\partial \rho}{\partial t}+(\mathbf{u}\cdot\nabla)\rho=0. \tag{2.5}$$

The presence of particles adds an extra force term, proportional to the velocity difference between particles and fluid and appears in equations of motion (2.1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The buoyancy force on the particles is neglected. Interparticle reactions are not considered for we assume that the distance between particles is quite large compared with their diameter.

The equations of motion and continuity for the particles, under the above approximations, are

$$mN\left[\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\mathbf{V}\right]=mN\mathbf{g}+KN(\mathbf{u}-\mathbf{V}), \tag{2.6}$$

$$\frac{\partial N}{\partial t}+\nabla\cdot(N\mathbf{V})=0, \tag{2.7}$$

where mN is the mass of the particles per unit volume.

Let $\mathbf{u}(u, v, w)$, $\mathbf{V}(l, r, s)$, $\delta\rho$, δp, and $\mathbf{h}(h_x, h_y, h_z)$ denote, respectively, the perturbations in fluid velocity $\mathbf{u}(0, 0, 0)$, particle velocity $\mathbf{V}(0, 0, 0)$, density ρ, pressure p, and the magnetic field $\mathbf{H}(H(z), 0, 0)$. Then the linearized perturbation equations of the fluid-particle layer are

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla \delta p + \mathbf{g}\,\delta\rho + \mu \nabla^2 \mathbf{u} + \left(\frac{\partial w}{\partial \bar{x}} + \frac{\partial \mathbf{u}}{\partial z}\right)\frac{d\mu}{dz}$$

$$+ \frac{\mu_e}{4\pi}\left[(\nabla \times \mathbf{h}) \times \mathbf{H} + (\nabla \times \mathbf{H}) \times \mathbf{h}\right] + KN(\mathbf{V} - \mathbf{u}), \tag{2.8}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{2.9}$$

$$\frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}), \tag{2.10}$$

$$\nabla \cdot \mathbf{h} = 0, \tag{2.11}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\mathbf{V} = \mathbf{u}. \tag{2.12}$$

In addition to Eqs. (2.8)–(2.12), we have the equation

$$\frac{\partial}{\partial t}\,\delta\rho = -w\left(\frac{d\rho}{dz}\right), \tag{2.13}$$

which ensures that the density of every particle remains unchanged as we follow it with its motion.

Analysing the disturbances into normal modes, we seek solutions whose dependence on x, y, and t is given by

$$\exp(ik_x x + ik_y y + nt), \tag{2.14}$$

where k_x, k_y are the horizontal components of the wave number, $k^2 = k_x^2 + k_y^2$, and n is the growth rate.

Using expression (2.14), Eqs. (2.8)–(2.13) give

$$[\rho(\tau n + 1) + mN]nu$$
$$= -(\tau n + 1)ik_x\,\delta p + \mu(\tau n + 1)(D^2 - k^2)u$$
$$+ (ik_x w + Du)(\tau n + 1)D\mu + \frac{\mu_e}{4\pi}(\tau n + 1)h_z(DH), \tag{2.15}$$

$$[\rho(\tau n + 1) + mN]nv$$
$$= -(\tau n + 1)ik_y\,\delta p + \mu(\tau n + 1)(D^2 - k^2)v$$
$$+ (ik_y w + Dv)(\tau n + 1)D\mu + \frac{\mu_e H}{4\pi}(\tau n + 1)(ik_x h_y - ik_y h_x), \tag{2.16}$$

$$
\begin{aligned}
&[\rho(\tau n+1)+mN]nw \\
&\quad = -(\tau n+1)D\,\delta p+\mu(\tau n+1)(D^2-k^2)w \\
&\quad + 2Dw(\tau n+1)(D\mu)+\frac{gw}{n}(\tau n+1)(D\rho)+\frac{\mu_e}{4\pi}(\tau n+1) \\
&\quad \times [H(ik_x h_z-Dh_x)-h_x(DH)],
\end{aligned} \tag{2.17}
$$

$$ik_x u+ik_y v+Dw=0, \tag{2.18}$$

$$ik_x h_x+ik_y h_y+Dh_z=0, \tag{2.19}$$

$$nh_x=ik_x Hu-w(DH), \tag{2.20}$$

$$nh_y=ik_x Hv, \tag{2.21}$$

$$nh_z=ik_x Hw, \tag{2.22}$$

where $\tau=m/K$.

Eliminating δp between Eqs. (2.15)–(2.17) and using Eqs. (2.18)–(2.22), we obtain

$$
\begin{aligned}
&n(\tau n+1)[D(\rho Dw)-k^2\rho w]+n[D(mN\,Dw)-k^2(mN)w] \\
&\quad -\mu(\tau n+1)(D^2-k^2)^2 w+\frac{gk^2}{n}(D\rho)(\tau n+1)w-(\tau n+1) \\
&\quad \times [D\{(D\mu)(D^2+k^2)w\}-2k^2(D\mu)(Dw)+(D\mu)(D^2+k^2)(Dw)] \\
&\quad +\frac{\mu_e k_x^2}{4\pi n}(\tau n+1)D(H^2Dw)-\frac{\mu_e H^2 k_x^2(\tau n+1)k^2}{4\pi n}w=0.
\end{aligned} \tag{2.23}
$$

3. Two Uniform Fluids Separated by a Horizontal Boundary

Consider the case of two uniform fluids of densities ρ_1 and ρ_2, viscosities μ_1 and μ_2, and magnetic fields $\mathbf{H}_1$ and $\mathbf{H}_2$ separated by a horizontal boundary at $z=0$. The subscripts 1 and 2 distinguish the lower and the upper fluids, respectively. In each of the two regions of constant ρ, μ, and $\mathbf{H}$, Eq. (2.23) reduces to

$$(D^2-k^2)(D^2-K^2)w=0, \tag{3.1}$$

where

$$K^2=k^2+\frac{n}{v}+\frac{nmN}{\rho v(\tau n+1)}+\frac{\mu_e k_x^2 H^2}{4\pi\rho v n}.$$

Since w must vanish both when $z \to -\infty$ (in the lower fluid) and $z \to +\infty$ (in the upper fluid), the general solution of Eq. (3.1) can be written as

$$w_1 = A_1 e^{+kz} + B_1 e^{+K_1 z} \qquad (z < 0) \tag{3.2}$$

$$w_2 = A_2 e^{-kz} + B_2 e^{-K_2 z} \qquad (z > 0), \tag{3.3}$$

where A_1, B_1, A_2, B_2 are constants of integration,

$$K_1 = \left[k^2 + \frac{n}{\nu_1} + \frac{nmN}{\rho_1 \nu_1 (\tau n + 1)} + \frac{\mu_e k_x^2 H_1^2}{4\pi \rho_1 \nu_1 n} \right]^{1/2}$$

and

$$K_2 = \left[k^2 + \frac{n}{\nu_2} + \frac{nmN}{\rho_2 \nu_2 (\tau n + 1)} + \frac{\mu_e k_x^2 H_2^2}{4\pi \rho_2 \nu_2 n} \right]^{1/2}.$$

For the sake of simplicity, we may consider that the Alfvén velocities of the two fluids are the same, so that

$$V_A^2 = \frac{\mu_e H_1^2}{4\pi \rho_1} = \frac{\mu_e H_2^2}{4\pi \rho_2}.$$

Thus the expressions for K_1 and K_2 take the forms

$$K_1 = \left[k^2 + \frac{n}{\nu_1} + \frac{nmN}{\rho_1 \nu_1 (\tau n + 1)} + \frac{k_x^2 V_A^2}{\nu_1 n} \right]^{1/2}, \tag{3.4}$$

$$K_2 = \left[k^2 + \frac{n}{\nu_2} + \frac{nmN}{\rho_2 \nu_2 (\tau n + 1)} + \frac{k_x^2 V_A^2}{\nu_2 n} \right]^{1/2}. \tag{3.5}$$

Integrating Eq. (2.23) across the interface at $z = 0$, we obtain

$$\begin{aligned} &\left\{ \left[\rho_2 - \frac{\mu_2}{n} (D^2 - k^2) \right] D w_2 \right\}_{z=0} - \left\{ \left[\rho_1 - \frac{\mu_1}{n} (D^2 - k^2) \right] D w_1 \right\}_{z=0} \\ &\quad + \frac{mN}{(\tau n + 1)} (D w_2 - D w_1)_{z=0} \\ &\quad + \frac{k_x^2 V_A^2}{n^2} (\rho_2 D w_2 - \rho_1 D w_1)_{z=0} \\ &= -\frac{g k^2}{n^2} (\rho_2 - \rho_1) w_0 - \frac{2k^2}{n} (\mu_2 - \mu_1)(Dw)_0. \end{aligned} \tag{3.6}$$

In addition to the condition (3.6), the boundary conditions to be satisfied at the interface $z=0$ are (Chandrasekhar [1, p. 432]

$$w,\ Dw,\ \mu(D^2+k^2)w, \tag{3.7}$$

must be continuous across an interface between two fluids.

Applying the boundary conditions (3.6) and (3.7) to the solutions given in (3.2) and (3.3), we obtain

$$A_1+B_1=A_2+B_2, \tag{3.8}$$

$$kA_1+K_1B_1=-kA_2-K_2B_2, \tag{3.9}$$

$$\mu_1[2k^2A_1+(K_1^2+k^2)B_1]=\mu_2[2k^2A_2+(K_2^2+k^2)B_2], \tag{3.10}$$

$$\begin{aligned}&\left[\frac{R}{2}+C-\rho_1-\frac{mN}{\tau n+1}-S\rho_1\right]A_1+\left[\frac{R}{2}+C\frac{K_1}{k}\right]B_1\\&\quad+\left[\frac{R}{2}-C-\rho_2-\frac{mN}{\tau n+1}-S\rho_2\right]A_2+\left[\frac{R}{2}-C\frac{K_2}{k}\right]B_2=0,\end{aligned} \tag{3.11}$$

where

$$R=\frac{gk(\rho_2-\rho_1)}{n^2},\qquad C=\frac{k^2}{n}(\mu_2-\mu_1),\qquad \text{and}\qquad S=\frac{k_x^2V_A^2}{n^2}.$$

Equations (3.8)–(3.11) can be written, in matrix notation, in the form of a single matrix equation

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ k & K_1 & k & K_2 \\ 2k^2\mu_1 & \mu_1(K_1^2+k^2) & -2k^2\mu_2 & -\mu_2(K_2^2+k^2) \\ \left\{\frac{R}{2}+C-\rho_1-\frac{mN}{\tau n+1}-S\rho_1\right\} & \left\{\frac{R}{2}+\frac{CK_1}{k}\right\} & \left\{\frac{R}{2}-C-\rho_2-\frac{mN}{\tau n+1}-S\rho_2\right\} & \left\{\frac{R}{2}-\frac{CK_2}{k}\right\} \end{bmatrix}\begin{bmatrix} A_1\\ B_1\\ A_2\\ B_2\end{bmatrix}=0. \tag{3.12}$$

The determinant of the linear system of equations which (3.12) represents must clearly vanish. The determinant can be reduced by subtracting the first column from the second, the third column from the fourth, and adding the first column to the third. By this procedure, we obtain

$$\begin{bmatrix} K_1 - k & 2k & K_2 - k \\ \left\{\rho_1 n + \dfrac{nmN}{\tau n + 1} + \dfrac{\rho_1 k_x^2 V_A^2}{n}\right\} & 2k^2(\rho_1 v_1 - \rho_2 v_2) & -\left\{\rho_2 n + \dfrac{nmN}{\tau n + 1} + \dfrac{\rho_2 k_x^2 V_A^2}{n}\right\} \\ \left\{\rho_1 + \dfrac{mN}{\tau n + 1} + \dfrac{C}{k}(K_1 - k) + S\rho_1\right\} & \left\{\dfrac{gk}{n^2}(\rho_2 - \rho_1) - \dfrac{2mN}{\tau n + 1} - (\rho_1 + \rho_2)(1 + S)\right\} & \left\{\rho_2 + \dfrac{mN}{\tau n + 1} - \dfrac{C}{k}(K_2 - k) + S\rho_2\right\} \end{bmatrix} = 0. \tag{3.13}$$

4. Discussion

Since the values of K_1 and K_2 involve square roots, the dispersion relation (3.13) is quite complicated. For mathematical simplicity, we make the assumptions that the kinematic viscosities of the two fluids are the same, i.e., $v_1 = v_2 = v$ and that the fluids are highly viscous. Under the above assumptions, we have

$$\begin{aligned} K &= k\left[1 + \frac{n}{vk^2} + \frac{nmN}{\rho v k^2(\tau n + 1)} + \frac{k_x^2 V_A^2}{k^2 v n}\right]^{1/2} \\ &= k + \frac{n}{2vk} + \frac{nmN}{2\rho v k(\tau n + 1)} + \frac{k_x^2 V_A^2}{2kvn}, \end{aligned} \tag{4.1}$$

so that

$$K_1 - k = \frac{n}{2vk} + \frac{nmN}{2\rho_1 v k(\tau n + 1)} + \frac{k_x^2 V_A^2}{2kvn}, \tag{4.2}$$

and

$$K_2 - k = \frac{n}{2vk} + \frac{nmN}{2\rho_2 v k(\tau n + 1)} + \frac{k_x^2 V_A^2}{2kvn}. \tag{4.3}$$

Substituting the values of $(K_1 - k)$ and $(K_2 - k)$ from the expressions (4.2) and (4.3) in the determinant (3.13) and simplifying it, after a little algebra, we obtain

$$\begin{aligned} &A_9 n^9 + A_8 n^8 + A_7 n^7 + A_6 n^6 + A_5 n^5 + A_4 n^4 \\ &\quad + A_3 n^3 + A_2 n^2 + A_1 n + A_0 = 0, \end{aligned} \tag{4.4}$$

where

$$
\begin{aligned}
A_9 &= \rho_1\rho_2\tau^3(\rho_1+\rho_2),\\
A_8 &= (\rho_1+\rho_2)\{2\rho_1\rho_2 vk^2\tau^3 + 3\rho_1\rho_2\tau^2 + mN\tau^2(\rho_1+\rho_2)\} + 2\rho_1\rho_2 mN\tau^2,\\
A_7 &= (\rho_1+\rho_2)\tau\{6\rho_1\rho_2 vk^2\tau + 2(\rho_1+\rho_2)vk^2 mN\tau + 3\rho_1\rho_2 + 3\rho_1\rho_2 k_x^2 V_A^2\tau^2\\
&\quad + 3m^2N^2 + 2(\rho_1+\rho_2)mN\} + \rho_1\rho_2\tau\{4mN + gk(\rho_1-\rho_2)\tau^2\},\\
A_6 &= (\rho_1+\rho_2)\{6\rho_1\rho_2 vk^2\tau + 4\rho_1\rho_2 vk^2 k_x^2 V_A^2\tau^3 + 4(\rho_1+\rho_2)vk^2 mN\tau\\
&\quad + 2vk^2m^2N^2\tau + \rho_1\rho_2 + 9\rho_1\rho_2 k_x^2V_A^2\tau^2 + 3m^2N^2 + (\rho_1+\rho_1)mN\\
&\quad + 2(\rho_1+\rho_2)mNk_x^2V_A^2\tau^2 + gk(\rho_1-\rho_2)mN\tau^2\} + 2\rho_1\rho_2 mN\\
&\quad + 2m^3N^3 + 4\rho_1\rho_2 mNk_x^2V_A^2\tau^2 + 3gk(\rho_1-\rho_2)\rho_1\rho_2\tau^2,\\
A_5 &= (\rho_1+\rho_2)\{2\rho_1\rho_2 vk^2 + 12\rho_1\rho_2 vk^2k_x^2V_A^2\tau^2 + 2(\rho_1+\rho_2)vk^2mN\\
&\quad + 2(\rho_1+\rho_2)vk^2mNk_x^2V_A^2\tau^2 + 2vk^2m^2N^2 + 9\rho_1\rho_2k_x^2V_A^2\tau\\
&\quad + 3\rho_1\rho_2(k_x^2V_A^2)^2\tau^3 + 4(\rho_1+\rho_2)mNk_x^2V_A^2\tau + 3k_x^2V_A^2m^2N^2\tau\\
&\quad + 2gk(\rho_1-\rho_2)mN\tau\} + 8\rho_1\rho_2mNk_x^2V_A^2\tau + 3gk(\rho_1-\rho_2)\rho_1\rho_2\tau\\
&\quad + gk(\rho_1-\rho_2)m^2N^2\tau + 2gk(\rho_1-\rho_2)\rho_1\rho_2k_x^2V_A^2\tau^3,\\
A_4 &= (\rho_1+\rho_2)\{12\rho_1\rho_2vk^2k_x^2V_A^2\tau + 2\rho_1\rho_2vk^2(k_x^2V_A^2)^2\tau^3\\
&\quad + 4(\rho_1+\rho_2)vk^2mNk_x^2V_A^2\tau + 3\rho_1\rho_2k_x^2V_A^2 + 9\rho_1\rho_2(k_x^2V_A^2)^2\tau^2\\
&\quad + 2(\rho_1+\rho_2)mNk_x^2V_A^2 + (\rho_1+\rho_2)mN(k_x^2V_A^2)^2\tau^2 + 3k_x^2V_A^2m^2N^2\\
&\quad + gk(\rho_1-\rho_2)mN + gk(\rho_1-\rho_2)mNk_x^2V_A^2\tau^2\} + 2\rho_1\rho_2mN(k_x^2V_A^2)^2\tau^2\\
&\quad + 4\rho_1\rho_2mNk_x^2V_A^2 + gk(\rho_1-\rho_2)\rho_1\rho_2 + gk(\rho_1-\rho_2)m^2N^2\\
&\quad + 6gk(\rho_1-\rho_2)\rho_1\rho_2k_x^2V_A^2\tau^2,\\
A_3 &= k_x^2V_A^2(\rho_1+\rho_2)\{4\rho_1\rho_2vk^2 + 6\rho_1\rho_2vk^2k_x^2V_A^2\tau^2\\
&\quad + 2(\rho_1+\rho_2)vk^2mN + 9\rho_1\rho_2k_x^2V_A^2\tau + \rho_1\rho_2(k_x^2V_A^2)^2\tau^3\\
&\quad + 2(\rho_1+\rho_2)mNk_x^2V_A^2\tau + 2gk(\rho_1-\rho_2)mN\tau\}\\
&\quad + \rho_1\rho_2k_x^2V_A^2\tau\{4mNk_x^2V_A^2 + gk(\rho_1-\rho_2)k_x^2V_A^2\tau^2 + 6gk(\rho_1-\rho_2)\},\\
A_2 &= k_x^2V_A^2(\rho_1+\rho_2)\{6\rho_1\rho_2vk^2k_x^2V_A^2\tau + 3\rho_1\rho_2k_x^2V_A^2\\
&\quad + 3\rho_1\rho_2(k_x^2V_A^2)^2\tau^2 + (\rho_1+\rho_2)mNk_x^2V_A^2 + gk(\rho_1-\rho_2)mN\}\\
&\quad + \rho_1\rho_2k_x^2V_A^2\{2mNk_x^2V_A^2 + 3gk(\rho_1-\rho_2)k_x^2V_A^2\tau^2 + 2gk(\rho_1-\rho_2)\},\\
A_1 &= \rho_1\rho_2(k_x^2V_A^2)^2(\rho_1+\rho_2)\{2vk^2 + 3k_x^2V_A^2\tau\}\\
&\quad + 3gk(\rho_1-\rho_2)\rho_1\rho_2(k_x^2V_A^2)^2\tau,\\
A_0 &= \rho_1\rho_2(k_x^2V_A^2)^2\{gk(\rho_1-\rho_2) + (\rho_1+\rho_2)k_x^2V_A^2\}.
\end{aligned}
$$

We have considered two uniform fluids of densities ρ_1 and ρ_2 separated by a horizontal boundary at $z=0$, where the subscripts 1 and 2 distinguish the lower and the upper fluids, respectively.

THEOREM 4.1. *If $\rho_1 < \rho_2$, then there exist infinitely many values of k for which Eq. (4.4) has at least one positive root.*

Proof. Consider the expression for A_0, which is given by

$$\begin{aligned} A_0 &= \rho_1 \rho_2 (k_x^2 V_A^2)^2 \{gk(\rho_1-\rho_2)+(\rho_1+\rho_2)k_x^2 V_A^2\} \\ &= \rho_1 \rho_2 (k_x^2 V_A^2)^2 \{-g(k_x^2+k_y^2)^{1/2} |\rho_1-\rho_2| + (\rho_1+\rho_2)k_x^2 V_A^2\}. \end{aligned}$$

Clearly, for any assumed positive value of k_y, if we choose

$$0 < k_x^2 < g^2(|\rho_1-\rho_2|)^2 + \frac{\{g^4(|\rho_1-\rho_2|)^4 + 4g^2(|\rho_1-\rho_2|)^2 k_y^2 (\rho_1+\rho_2)^2 V_A^4\}^{1/2}}{2(\rho_1+\rho_2)^2 V_A^4},$$

then, A_0 is negative. This implies that the product of the roots of Eq.(4.4) in the complex n-plane is positive. Further, since the complex roots occur in pairs, we must have at least one positive root for n, the degree of Eq. (4.4) being nine, which is odd. The possibility of having three or five or seven or nine real and positive roots for n is also not ruled out from here. Theorem (4.1) however establishes an important effect in the system, namely that the system is unstable for sufficiently long waves, however large the applied magnetic field may be, provided the initial configuration is top heavy density wise.

For the potentially stable configuration ($\rho_1 > \rho_2$), it can be shown that the system is stable in the presence of suspended particles.

THEOREM 4.2. *If $\rho_1 > \rho_2$, then there exists no positive real root or complex root with positive real part.*

Proof. For $\rho_1 > \rho_2$, all the coefficients of Eq. (4.4) are positive. Therefore, all the roots of Eq.(4.4) are either real and negative or there are complex roots (which occur in pairs) with negative real parts and the rest negative real roots. The system is therefore stable in each case. Hence, Theorem (4.2) implies that the potentially stable system remains stable for the case of two superposed fluids in the presence of suspended particles.

We conclude therefore that the system is stable for stable configuration and unstable for unstable configuration in the presence of a variable magnetic field and suspended particles. This is in contrast to the thermal instability (Bénard convection) problem where the suspended particles have a destabilizing effect.

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